演習問題 5 解答

問題 5.1

5.5 節参照のこと

問題 5.2

5.7 節参照のこと

問題 5.3

0.833 mm (円筒の変形は 9.09×10^{-3} mm にすぎない)

問題 5.4

5.6 節参照のこと

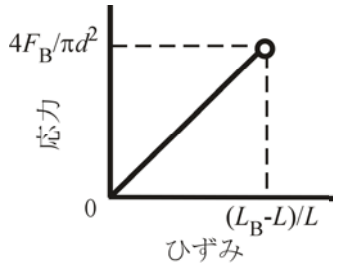
問題 5.5

(1) 真の値に対して $\dfrac{2E_0}{E_1}\dfrac{A_0L_1}{A_1L_0}$ 大きい値となる (2) $\begin{aligned}&2\sigma_2A_2+\sigma_1A_1=F\\ &\dfrac{\sigma_2(L_0+2L_1)}{E_2}=\dfrac{\sigma_1L_1}{E_1}+\dfrac{\sigma_0L_0}{E_0}\end{aligned}$

(3) $\sigma_0=\dfrac{F}{A_0}\dfrac{A_0A_aE_0E_aL_b}{A_0A_aE_0E_aL_b+4A_0A_bE_0E_bL_a+2A_aA_bE_aE_bL_0}$

問題 5.6

(1) $E=\dfrac{4F_B}{\pi d^2}\dfrac{L}{L_B-L}$

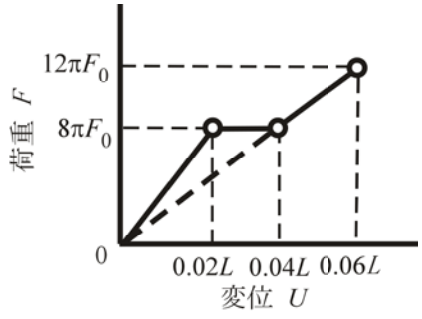


(2) $\dfrac{P_0}{F_B}\left(\dfrac{d^2}{d_1^2}\dfrac{L_1}{L}+\dfrac{d^2}{d_2^2}\dfrac{L_2}{L}+\dfrac{d^2}{d_3^2}\dfrac{L_3}{L}\right)(L_B-L)$ (3) $\dfrac{d_1^2}{d^2}F_B$

問題 5.7

(1) $E_I=\dfrac{200F_0}{d^2},\quad E_{II}=\dfrac{50F_0}{d^2}$ (2) $P=\dfrac{400\pi F_0}{L}U$

(3)



問題 5.8

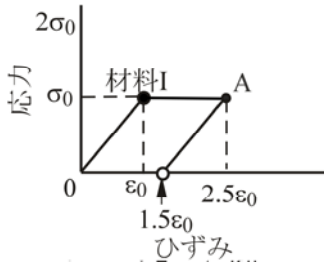
(1) $\dfrac{PL}{2A_aE_a + A_bE_b}$　　(2) $\dfrac{PL}{2A_aE_a}$

問題 5.9

(1) $E_A = \dfrac{4P_{A1}L_A}{\pi D_A^2\left(L_{A1} - L_a\right)}$, $E_B = \dfrac{4P_{B2}L_{B1}}{\pi D_B^2\left(L_{B2} - L_{B1}\right)}$　　(2) $\dfrac{4L_AL_BF}{\pi\left(D_A^2E_AL_B + D_B^2E_BL_A\right)}$

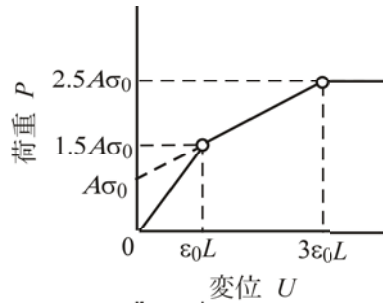
問題 5.10

(1) $E_I = \dfrac{\sigma_0}{\varepsilon_0}$, $E_{II} = \dfrac{\sigma_0}{2\varepsilon_0}$



(2) $\sigma_1 = \dfrac{2P}{3A}$, $\sigma_1 = -\dfrac{P}{3A}$

(3)



問題 5.11

(1) $P_A = \dfrac{A}{A_0}P_{A3}$, $P_B = \dfrac{A}{A_0}P_{B2}$　　(2) $\sigma_B = \dfrac{L_1 - L}{L}E_A$　　(3) $\sigma_B = \dfrac{L_1 - L}{L}\dfrac{E_AE_B}{E_A + E_B}$

問題 5.12

(1-a) $\dfrac{\sigma_A}{E_A} = \dfrac{\sigma_B}{E_B}$　　(1-b) $u_1 = \dfrac{L}{A_AE_A + A_BE_B}P_1$

(2-a) $P_2 = \dfrac{A_AE_A}{L}u_2$　　(2-b) $P_2 = \dfrac{LA_BE_B + \left(L - C\right)A_AE_A}{L\left(L - C\right)}u_2 - \dfrac{CA_BE_B}{L - C}$

問題 5.13

(1) $E_A = \dfrac{4xP_0}{\pi D^2\left(a - 1\right)}$, $E_B = \dfrac{8yP_0}{\pi D^2\left(b - 2\right)}$　　(2) $\sigma_B = \dfrac{\left(a - 1\right)y}{\left(b - 2\right)x + \left(a - 1\right)y}\dfrac{4F}{\pi D^2}$

(3) $F_1 = \dfrac{\left(b - 2\right)x + \left(a - 1\right)y}{\left(b - 2\right)x}xP_0$　　(4) $u_B = \dfrac{\left\{\left(b - 2\right)x + \left(a - 1\right)y\right\}L}{y}$

問題 5.14

(1) $E=\dfrac{P_1}{\pi d_0^2/4}\dfrac{a_0}{a_1-a_0}=\dfrac{4P_1a_0}{\pi d_0^2(a_1-a_0)}$ ， $\sigma_B=\dfrac{4P_1}{\pi d_0^2}$ ， $d=d_0-\nu\dfrac{a_1-a_0}{a_0}\dfrac{P}{P_1}d_0$

(2) $a_0+\dfrac{f_a}{\pi d_0^2/4}\dfrac{1}{E}a_0=b_0+\dfrac{f_b}{k}$

(3) $L_0=\dfrac{4b_0k+\pi d_0^2E}{\pi d_0^2E+4a_0k}a_0$

(4) $k=\dfrac{\pi\sigma_B d_0^2E}{4E(b_0-a_0)+4a_0\sigma_B}$

問題 5.15

(1) $P_1=\sigma_yA,\quad P_2=\sigma_BA,\quad L_1=L+\dfrac{\sigma_yL}{E}$

(2) $U=\dfrac{F}{3}\dfrac{1}{A}\dfrac{L}{E}=\dfrac{FL}{3AE}$

(3) $F(A)=3\sigma_yA$， $F(B)=\dfrac{9\sigma_yA}{4}=2.25\sigma_yA$， $F=3\sigma_yA$

問題 5.16

(1) 縦弾性係数 σ_1/ε_1，降伏応力 σ_1，引張強度 σ_2 降伏荷重 σ_1A，
最大荷重 σ_2A

(2) $2A_0$ の部分 $\sigma=0$， A_0 の部分 $\sigma=\dfrac{F}{A_0}$

(3) $x=L_0$ において破断する 応力集中のため

(4) L_1 の部分の伸び $\lambda=\dfrac{P_1}{A_0E}L_1$, L_0 の部分の伸び $\lambda=\dfrac{P_0}{2A_0E}L_0$

力のつり合い $P_1+P_0=0$

$L=L_1+\dfrac{-P_0}{A_0E}L_1=L_0+\dfrac{P_0}{2A_0E}L_0$ ， $L=\dfrac{3L_1L_0}{2L_1+L_0}$

問題 5.17

(1) 縦弾性係数: $\dfrac{P_1}{A_0}\dfrac{L_0}{L_1-L_0}$， 降伏応力: $\dfrac{P_1}{A_0}$， 引張強度: $\dfrac{P_2}{A_0}$， $L_A-\dfrac{P_A}{P_1}(L_1-L_0)$

(2) $\sigma_I=\dfrac{1}{3}\alpha E\Delta T$， $\sigma_{II}=\dfrac{4}{3}\alpha E\Delta T$

(3) $\Delta T<\dfrac{3\sigma_Y}{4S\alpha E\Delta T}$

問題 5.17

(1) 縦弾性係数: $\dfrac{P_1}{A_0}\dfrac{L_0}{L_1-L_0}$， 降伏応力: $\dfrac{P_1}{A_0}$， 引張強度: $\dfrac{P_2}{A_0}$， $L_A-\dfrac{P_A}{P_1}(L_1-L_0)$

(2) 力のつり合い $\sigma_I 4A = \sigma_{II} A$　変形の拘束条件 $-\alpha\cdot\Delta T\cdot 2L + \frac{\sigma_I}{E}2L + \frac{\sigma_{II}}{E}L = 0$，$\sigma_{II} = 4\sigma_I$，

$2\sigma_I + 4\sigma_I = 2\alpha E\Delta T$，$\sigma_I = \frac{1}{3}\alpha E\Delta T$，$\sigma_{II} = \frac{4}{3}\alpha E\Delta T$

(3) $\sigma_{II} = \frac{4}{3}\alpha E\Delta T = \frac{\sigma_Y}{S}$，$\Delta T < \frac{3\sigma_Y}{4S\alpha E\Delta T}$

問題 5.18

(1) $\sigma_I = \frac{F_0}{\pi a^2}$，$\sigma_{II} = 0$

(2) $u_A = \frac{F}{\pi E a^2}16a = \frac{16F}{\pi Ea}$

(3) $u_A = \frac{R_1}{\pi E a^2}16a + \frac{R_1 - F}{2\pi E a^2}32a = 0$，$R_1 = -R_2 = \frac{F}{2}$　$\sigma_I = \frac{F}{2\pi a^2}$，$\sigma_{II} = -\frac{F}{2\pi a^2}$

(4) $u_B = \frac{F}{2\pi E a^2}16a = \frac{8F}{\pi Ea^2}$

(5)
$u_A = \frac{R_1}{\pi E a^2}16a + \frac{R_1 - F}{2\pi E a^2}32a = Ca$，$R_1 = \frac{\pi CEa^2}{32} + \frac{F}{2}$　$R_2 = \frac{\pi CEa^2}{32} - \frac{F}{2}$

$\sigma_I = \left(\frac{\pi CEa^2}{32} + \frac{F}{2}\right)\frac{1}{\pi a^2} = \frac{CE}{32} + \frac{F}{2\pi a^2}$，$\sigma_{II} = \frac{CE}{32} - \frac{F}{2\pi a^2}$

問題 5.19

(1) $\sigma_A = \frac{1}{\pi\alpha^2 R^2}F_1$，$\sigma_B = \frac{1}{\pi R^2}F_1$

(2) $\sigma_A = \frac{E}{L}\frac{1}{1+\alpha^2}u_2$，$\sigma_B = \frac{E}{L}\frac{\alpha^2}{1+\alpha^2}u_2$

(3) $\sigma_A = \frac{1}{\pi R^2}\frac{1}{1+\alpha^2}F_2$，$\sigma_B = \frac{1}{\pi R^2}\frac{1}{1+\alpha^2}F_3$

(4) α に関係なく棒 A を先に降伏させることができない.