

Answers of Problems 5

5.1

Principal stresses: $\sigma_1 = 200 \text{ MPa}$, $\sigma_2 = 100 \text{ MPa}$, principal shear stress: $\tau_{1,2} = \pm 50 \text{ MPa}$

5.2

Principal stresses: $\sigma_1 = 240 \text{ MPa}$, $\sigma_2 = 0 \text{ MPa}$, principal shear stress: $\tau_{1,2} = \pm 120 \text{ MPa}$

5.3

Omitted.

5.4

$$(1) \sigma_x = \frac{R}{2h}P, \quad \sigma_y = \frac{R}{h}P, \quad \tau_{xy} = 0 \quad \varepsilon_x = \frac{RP}{2hE}(1-2\nu), \quad \varepsilon_y = \frac{RP}{2hE}(2-\nu), \quad \gamma_{xy} = 0$$

$$(2) \varepsilon_1 = \frac{RP}{2hE}(1-2\nu), \quad \varepsilon_2 = \frac{RP}{2hE}(2-\nu), \quad \gamma_{1,2} = \pm \frac{RP}{hE}(1+\nu)$$

5.5

Principal stresses: $\sigma_1 = 22.7 \text{ MPa}$, $\sigma_2 = -8.7 \text{ MPa}$. Principal shear stress: $\tau_{1,2} = \pm 15.7 \text{ MPa}$

5.6

$$\sigma_1 = \frac{290}{d^2} \text{ kPa}, \quad \sigma_2 = -\frac{36}{d^2} \text{ kPa}$$

5.7

Principal shear stress: $\tau_{1,2} = \pm 1.45 \text{ MPa}$, normal stress on the plane of principal shear stress:

$$\sigma = 8.5 \text{ MPa}$$

5.8

$$(1) E = \frac{32(1+\nu)}{\pi d^3 \gamma_0}$$

$$(2) \sigma_x = \frac{2\sqrt{2}}{\pi d^3}(d+4L)F, \quad \sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$$

5.9

$$(1) \sigma_x = \frac{40F}{\pi d^2}, \quad \sigma_y = 0, \quad \tau_{xy} = \frac{4F}{\pi d^2}$$

$$(2) \sigma_{1,2} = \frac{4F}{\pi d^2}(5 \pm \sqrt{26})$$

5.10

$$(1) \sigma_x = \frac{3F}{a}, \quad \sigma_y = \frac{F}{a}, \quad \tau_{xy} = \frac{F}{a}, \quad -\frac{F}{a}$$

$$(2) \sigma_1 = (2 + \sqrt{2})\frac{F}{a}, \quad \sigma_2 = (2 - \sqrt{2})\frac{F}{a}, \quad 22.5^\circ = \frac{\pi}{8}$$

$$(3) \varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} = (3 - \nu)\frac{F}{aE}$$

5.11

(1) bending moment: $3FR$, torsional moment: $6FR$, tensile force: 0

$$(2) \sigma_x = \frac{12F}{\pi R^2}, \quad \sigma_y = 0, \quad \tau_{xy} = \frac{12F}{\pi R^2}$$

$$(3) \sigma_1 = \frac{6(1+\sqrt{5})F}{\pi R^2}, \quad \sigma_2 = \frac{6(1-\sqrt{5})F}{\pi R^2}$$

$$(4) \sigma_1 = \frac{12F}{\pi R^2}, \quad \sigma_2 = -\frac{12F}{\pi R^2}$$

5.12

$$(1) \sigma_x = \frac{RP}{2h}, \quad \sigma_y = \frac{RP}{h}, \quad \tau_{xy} = 0$$

$$(2) \sigma_2 = \frac{RP}{2h}, \quad \sigma_1 = \frac{RP}{h}, \quad \tau_1 = \frac{RP}{4h}$$

$$(3) \varepsilon_0 = \frac{PR(1-2\nu)}{2Eh}$$

$$(4) P = \frac{8Eh}{R(5-7\nu)}\varepsilon_1$$

5.13

$$(1) \sigma_x = \frac{d}{4t}P, \quad \sigma_y = \frac{d}{2t}P, \quad \tau_{xy} = 0$$

$$(2) \sigma_1 = \frac{Pd}{2t}, \quad \sigma_2 = \frac{Pd}{4t}$$

$$(3) \tau_1 = \frac{Pd}{8t}$$

$$(4) \varepsilon_x = \frac{1}{E} \frac{Pd}{4t} - \nu \frac{Pd}{E} \frac{1}{2t} = \frac{Pd}{4tE} (1-2\nu)$$

5.14

$$(1) \sigma_y = \frac{2F}{ah}$$

$$(2) \gamma_{xy} = \frac{15F}{2Eah}$$

$$(3) \sigma_1 = \frac{F}{ah}(5+3\sqrt{2}), \quad \sigma_2 = \frac{F}{ah}(5-3\sqrt{2})$$

$$(4) \sigma_x' = \frac{8F}{ah}$$

$$(5) \varepsilon_x' = \frac{15F}{Eah}$$

5.15

$$(1) \varepsilon_x = \frac{(4-8\nu)A}{E}$$

$$(2) \sigma_1 = 10A, \quad \sigma_2 = 2A$$

$$(3) A = \frac{P_0 R}{10H}$$

$$(4) P_0 = \frac{10EH}{(4-8\nu)R} \varepsilon_0$$